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Embedded Systems

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Note:

The given solution is only a proposal. For correctness, completeness, or understandability, no responsability is taken.

Task 1 : Real-Time Scheduling (maximal 42 points)

1.1: Fixed-Priority Scheduling (maximal 17 points)

A periodic task-set as shown in Table 1 is to be scheduled on a processor. The first release of each task occurs at time zero.

Task	Computation Time	Period	Deadline
τ_1	2	5	4
τ_2	1	4	3
τ_3	2	8	8

Table 1: A periodic task-set

(a) (3 points) Construct the scheduling diagram under Deadline Monotonic (DM) preemptive scheduling.

Sample solution:

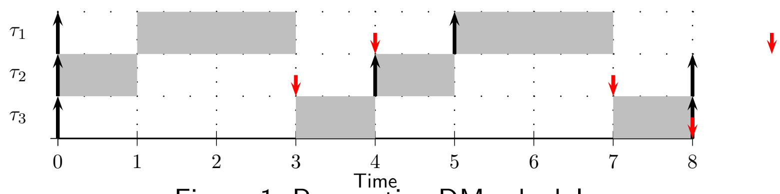


Figure 1: Preemptive DM schedule

The priorities are $\tau_2 > \tau_1 > \tau_3$.

(b) (2 points) Construct the scheduling diagram under DM non-preemptive scheduling (i.e. once started, any task has to run to completion before yielding the processor to the other tasks).

Sample solution:

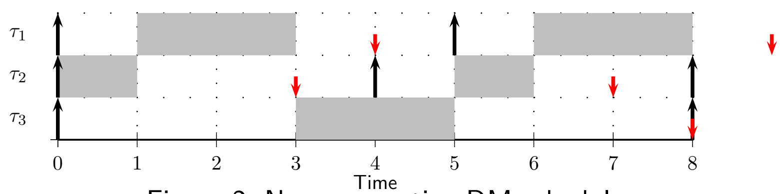


Figure 2: Non-preemptive DM schedule

The priorities are the same as before.

(c) (9 points) Assume preemptive scheduling and DM priority assignment.

(1) (3 points) Does the task-set pass the sufficient schedulability test?

Sample solution: No, it does not, as the following does not hold:

$$\begin{gathered} 2/4 + 1/3 + 2/8 \leq 3(2^{1/3} - 1) \\ 1.083 \leq 0.780 \end{gathered} \tag{1}$$

(2) (6 points) Does the task-set pass the necessary and sufficient schedulability test?

Sample solution: Yes it does, as every task meets its deadline:

τ_2: $R_2 = 1$.
τ_1: $R_1 = 2 + \lceil \frac{R_1}{4} \rceil \times 1$, $R_1 = 3$.
τ_3: $R_3 = 2 + \lceil \frac{R_3}{5} \rceil \times 2 + \lceil \frac{R_3}{4} \rceil \times 1$, $R_3 = 8$.

(d) (3 points) Assume non-preemptive fixed-priority scheduling. In the worst-case, how many lower priority jobs can delay the execution of one job? Justify your answer.

Sample solution:
At most one lower priority job.

We can prove this by contradiction. Suppose there are more than one lower priority jobs that could interfere with the execution of one job; then, when the jobs switch, the system would choose instead of the rest of the lower priority jobs, the job under analysis. Contradiction.

1.2: EDF Scheduling (maximal 16 points)

A periodic task-set as shown in Table 2 is to be scheduled on a preemptive processor under Earliest Deadline First (EDF). For a periodic task τ_i, we denote its computation time with C_i, period with T_i, and relative deadline with D_i. We assume that the first release of each task occurs at time zero.

Task	Computation Time	Period	Deadline
τ_1	1	4	3
τ_2	1	2	2
τ_3	2	8	8

Table 2: A periodic task-set

(a) (2 points) Construct the preemptive EDF scheduling diagram.

Sample solution: Notice that over interval [6,8], the order can be swapped as two jobs have the same deadline.

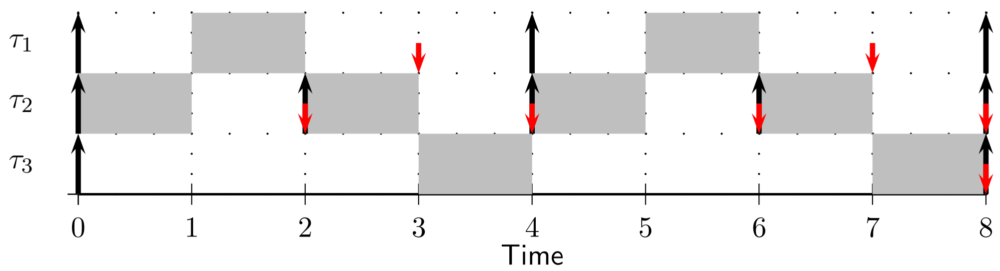


Figure 3: EDF schedule

(b) (2 points) A periodic task-set with constrained deadlines (i.e. $D_i \le T_i, \forall \tau_i$) is schedulable if $\sum_i \frac{C_i}{D_i} \le 1$. Does the task-set pass this sufficient schedulability test?

Sample solution: It does not, as $1/3 + 1/2 + 2/8 > 1$.

(c) (12 points) The demand bound (db) of a periodic task τ_i for any time interval of length Δ is defined as

$$\text{db}(\tau_i, \Delta) = \max\left\{\left\lfloor \frac{\Delta - D_i}{T_i} \right\rfloor + 1,\ 0\right\} \cdot C_i.$$

It is known that a set of such periodic tasks $\{\tau_i \mid 1 \le i \le n\}$ (with arbitrary first release times) is schedulable on a preemptive processor under EDF if and only if

$$\forall \Delta \ge 0, \sum_{i=1}^{n} \text{db}(\tau_i, \Delta) \le \Delta.$$

(1) (9 points) Plot the total system db as a function of Δ for the task-set in Table 2. Based on the above demand bound test, is the system schedulable under EDF?

Sample solution: In [0,8], the demand bound test condition is satisfied. After that, the system just repeats (with a hyper-period of 8). Hence, the system is schedulable.

Note, in the Figure we have three lines to represent individual db's of tasks, the thin red lines represents the sum of these or total db, and the thick red line is the supply.

Sample solution:

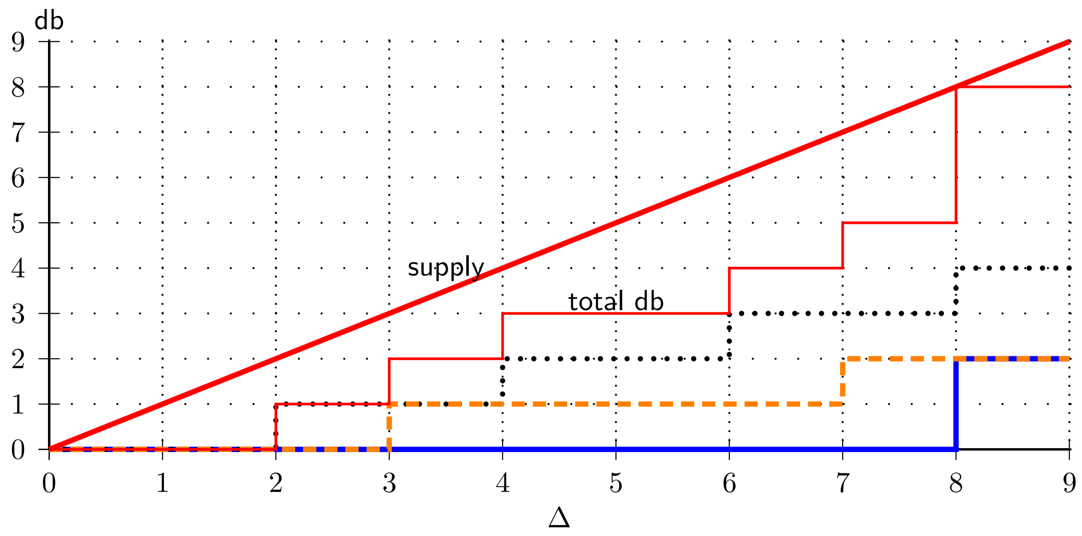


Figure 4: System db

(2) (3 points) Explain in a few sentences why passing the demand bound test is a necessary schedulability condition.
Hint: You can think about what db represents.

Sample solution: For an interval length of Δ, the demand bound function of a task represents the maximum workload that *must* be finished in any interval of length Δ. Therefore, if the demand bound test is not passed, no scheduling algorithm can meet all task deadlines.

1.3: Server Scheduling (maximal 9 points)

A periodic task-set as shown in Table 3 is to be scheduled on a preemptive processor. In addition, a job J with unknown arrival time and computation time of 5 units needs to be scheduled on the processor.

(a) (3 points) Assume Rate Monotonic (RM) scheduling for the periodic tasks, while job J is served by a polling server with period $T_S = 3$ and computation time $C_S = 0.5$. What is the smallest relative deadline that J can always satisfy?

Task	Computation Time	Period	Deadline
τ_1	1	4	4
τ_2	2	5	5

Table 3: A periodic task-set

Sample solution: $R_J = (1 + \lceil \frac{5}{0.5} \rceil) \times 3 = 33.$

(b) (6 points) Assume EDF scheduling for the periodic tasks, while job J is served by a total bandwidth server. What is the smallest relative deadline that J can always satisfy such that the periodic tasks are also schedulable?

Sample solution: $R_J = 0 + \frac{5}{U_{server}} \geq \frac{5}{1-1/4-2/5} = 14.3.$

Task 2 : Communication & Low Power Design (maximal 37 points)

2.1: TDMA & Energy Harvesting (maximal 11 points)

Four sensor nodes (S_1, S_2, S_3, S_4) transmit messages to a host H over a wireless network with the topology depicted in Figure 5.

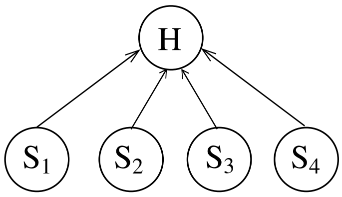


Figure 5: Network Topology

Communication occurs during frames that repeat periodically. Each frame is composed of four slots of equal duration T_{slot}, with one slot for each sensor. Sensors transmit messages during slots according to a Time Division Multiple Access (TDMA) protocol. The radio of the host and each sensor supports a data rate within the range of [1,10] Mbps (10^6 bits per second), and the propagation delay is negligible.

(a) (5 points) Assume each sensor S_i is assigned a slot duration $T_{\mathrm{slot}} = 50$ ms. Each sensor makes periodically available 125 kb (1 kb $= 10^3$ bits) with a period of 250 ms. What is the minimum data rate given that the maximum allowed latency is 200 ms?

Definition: Latency is the time between making data available for transmission at the sensor and receiving the data at the host.

Sample solution:
Since T_{slot} is already set, a maximum latency of 200 ms implies that each node transmits their 125 kb during one frame. The minimum data rate is then $D = 125kb/50ms = 2.5Mbps$.

(b) (6 points) Consider the TDMA schedule shown in Figure 6, which includes additional time $T_{\text{processing}}$ for the host to process the received messages. During $T_{\text{processing}}$, the host must compute for 10^5 cycles. The host processor can execute at a clock frequency F from the discrete set $\{5, 10, 20\}$ MHz. Assume that frequency is proportional to voltage ($F \sim V_{\text{dd}}$) and the power P satisfies $P \sim V_{\text{dd}}^2 * F$.

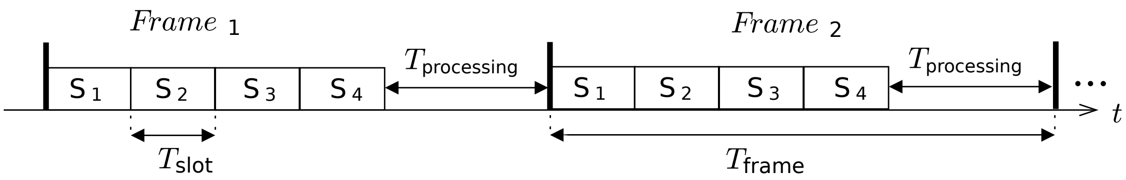


Figure 6: TDMA schedule including host processing time.

If $T_{\text{slot}} = 10$ ms and $T_{\text{frame}} \leq 50$ ms, find the host frequency which minimizes the **energy** required for processing. Justify your answer.

Sample solution:
Given the above inequality, it follows that $T_{processing} \leq 10$ ms. For frequencies $F = \{5M, 10M, 20M\}$, their processing time is $T_{processing} = \{20ms, 10ms, 5ms\}$, respectively. Hence, $F = 5M$ is not a valid frequency.

The energy consumed when F=10M is $E_{10M} \sim 10\text{MHz} * V_{dd,10M}^2 * 10\text{ms} = 10^5 * V_{dd,10M}^2$.

The energy consumed when F=20M is $E_{20M} \sim 20\text{MHz} * V_{dd,20M}^2 * 5\text{ms} = 10^5 * V_{dd,20M}^2$.

Given that $F \sim V_{dd}$, it follows that $V_{dd,10M} < V_{dd,20M}$. Consequently, F=10M consumes the least energy.

2.2: Dynamic Voltage Scaling (maximal 14 points)

Consider a processor whose dynamic power dissipation when running with a clock frequency f in Hz, is given by $P_{\text{dynamic}} = \left(\frac{f}{10^6 Hz}\right)^3$ mW. It is assumed that the processor has negligible static and leakage power dissipation.

The processor must execute the following task-set:

Task	τ_1	τ_2	τ_3
Arrival Time (ms)	0	2	4
Absolute Deadline (ms)	6	5	9
Cycles (x10^3)	3	9	3

Table 4: Task set.

Assume the clock frequency of the processor can be selected from a continuous range of frequencies. Apply the *online* YDS algorithm and plot the schedule in Figure 7. How much energy does the processor consume to complete all tasks?

Sample solution:

Apply the online YDS algorithm using intensity function $G[z_1, z_2] = \frac{\sum_{i \in V'} c_i}{z_2 - z_1}$.

Step 1:
$\tau_1 : G[0, 6] = 3/6$
Schedule task τ_1 @ 0.5 MHz

Step 2:
$\tau_2 : G[2, 5] = 9/3$
$\tau_1 : G[2, 6] = 11/4$
Schedule task τ_2 @ 3 MHz

Step 3:
$\tau_2 : G[4, 5] = 3/1$
$\tau_1 : G[4, 6] = 5/2$
$\tau_3 : G[4, 9] = 8/5$
Schedule task τ_2 @ 3 MHz

Step 4:
$\tau_1 : G[5, 6] = 2/1$

Schedule task τ_1 @ 2 MHz

Step 5:
$\tau_3 : G[6, 9] = 3/3$
Schedule task τ_3 @ 1 MHz

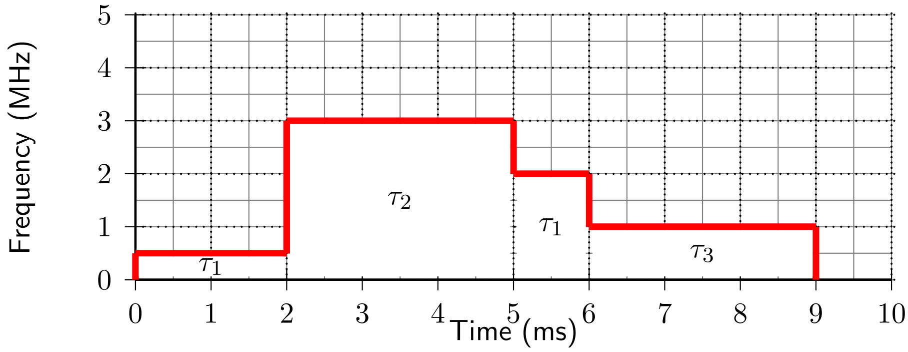


Figure 7: Online YDS Schedule.

To calculate the energy, different methods can be used. One is to convert the above figure to power, given the $P_{dynamic}$ equation, and calculate the area underneath the curve. Since $P_{avg} = 0.125 * (2/9) + 27 * (3/9) + 8 * (1/9) + 1 * (3/9) = 10.25mW$, it follows that $E_{consumed} = P_{avg} * T = 10.25mW * 9ms = 92.25uJ$.

□

2.3: Dynamic Power Management (maximal 12 points)

Consider a processor supporting two modes: *IDLE* and *ACTIVE*. The power dissipation is $P_{\text{idle}} = 5$ mW and $P_{\text{active}} = 50$ mW, respectively. The processor must schedule the following periodic task-set:

Task	τ_1	τ_2	τ_3
Period (ms)	10	10	10
Arrival of First Task (ms)	0	3	5
Relative Deadline (ms)	3	7	10
Execution Time (ms)	1	1	2

Table 5: Periodic task-set.

Definition: A workload-conserving scheduler always executes a task when the ready queue is not empty.

Definition: The average power dissipation between time 0 and T is given by $P_{\text{avg}} = \frac{1}{T}\int_0^T P(\tau)d\tau$.

(a) (6 points) Assuming zero energy and time overhead between IDLE and ACTIVE mode, plot the power dissipation during **two periods** for a workload-conserving schedule using Figure 8. The processor is in IDLE mode at time 0. What is the average power P_{avg} between time 0 ms and 20 ms?

Sample solution:
We can calculate the average power of one period by taking into account the time in each system mode. Since the schedule in both periods are the same, the average of two periods is the same as the average of one: $P_{avg} = P_{active} * 4/10 + P_{idle} * 6/10 = 23mW$.

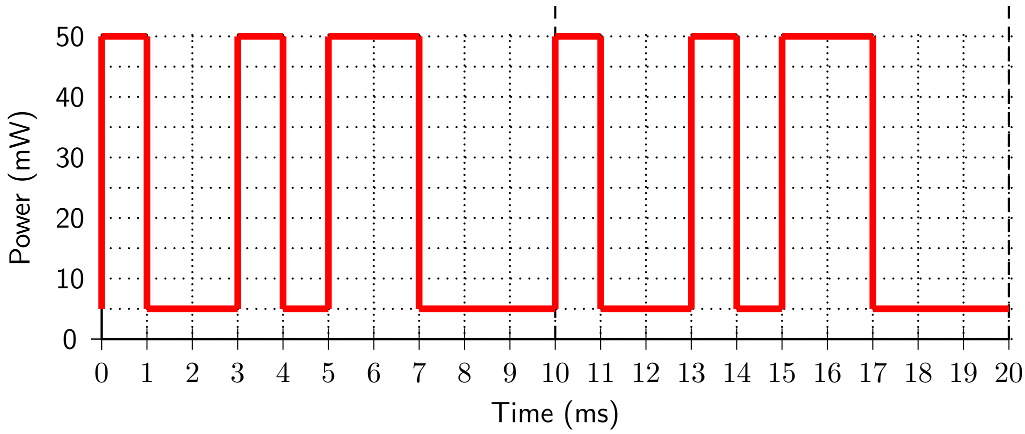


Figure 8: Solution for workload-conserving scheduler.

□

(b) (6 points) Now assume that there is a third mode called SLEEP, with $P_{\mathrm{sleep}} = 0$ mW. The system modes and allowed transitions, along with their energy and time overheads, are shown in Figure 9. Power dissipation during a mode transition is **constant**. A transition to ACTIVE mode occurs when there is at least one task in the ready queue. Plot the power dissipation during two periods for an **energy-minimizing**, **workload-conserving** schedule using Figure 10. The processor is in SLEEP mode at time 0. What is the average power P_{avg} between time 0 ms and 20 ms?

Figure 9: System modes and allowed transitions.

Sample solution:
First, we have to calculate the break-even time $t_{bev} \geq 7.5\mu J/5mW = 1.5ms$. Since the gaps between tasks τ_1 and τ_2 are only 1ms long, the system should stay in IDLE during these gaps. After task τ_3, the 3ms gap is greater the break-even time, meaning the system should enter SLEEP mode. Lastly, the average power during the switch from SLEEP to ACTIVE, $P_{switch} = E_{switch}/t_{switch} = 7.5mW$. Now, we can calculate the average power of one period by taking into account the time in each system mode. Since the schedule in both periods are the same, the average of two periods is the same as the average of one: $P_{avg} = P_{active} * 4/10 + P_{switch} * 1/10 + P_{idle} * 2/10 + P_{sleep} * 3/10 = 21.75mW$.

□

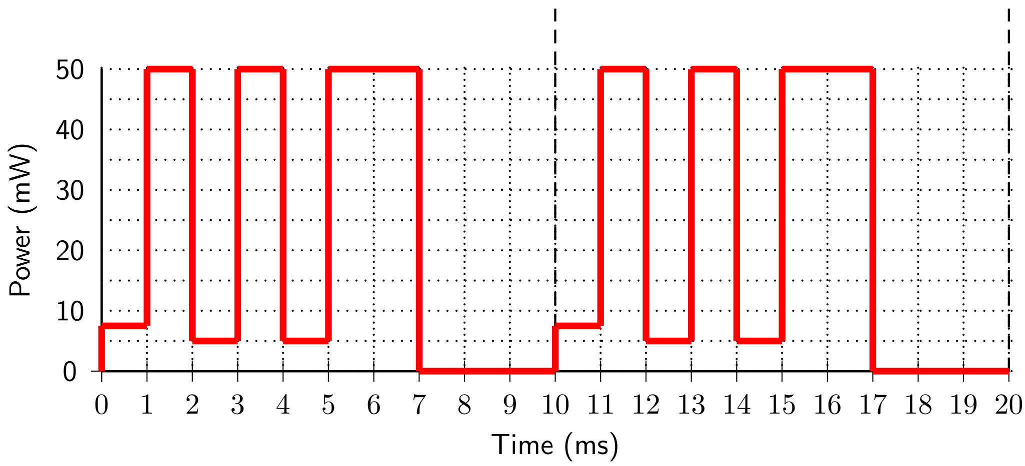


Figure 10: Power dissipation using an energy-minimizing workload-conserving scheduler

Task 3 : Architecture Synthesis (maximal 41 points)

3.1: Architecture Synthesis Fundamentals (maximal 7 points)

Mark the following statements as *true* or *false* and provide a one sentence explanation.

- (1 point) In a sequence graph, a LOOP node is considered an *operation or task* node.

  Sample solution:
  ☐ True ☒ False
  Explanation: Sequence graphs have operation or tasks nodes, and hierarchy nodes. A LOOP node is the latter.

  ☐

- (1 point) *ALAP* (as late as possible) scheduling provides a schedule with maximal latency, when *limited* resources are available.

  Sample solution:
  ☐ True ☒ False
  Explanation: ALAP scheduling produces minimal latency, assuming unlimited resources.

  ☐

- (1 point) A *dependence graph* does not specify a schedule.

  Sample solution:
  ☒ True ☐ False
  Explanation: A dependence graph specifies partial order between operations, not the schedule.

  ☐

- (1 point) In a *marked graph*, tokens on edges can be interpreted as data stored in a LIFO (last in, first out) queue.

  Sample solution:
  ☐ True ☒ False
  Explanation: Tokens correspond to data stored in FIFO queues.

☐

- (1 point) A *marked graph* can be implemented in hardware.

  Sample solution:

  ☒ True ☐ False

  Explanation: There are different possibilities of marked graphs implementation, both in hardware and software.

☐

- (1 point) In *List* scheduling, node priorities change as the algorithm executes through different time steps.

  Sample solution:

  ☐ True ☒ False

  Explanation: In list scheduling, priorities are determined a priori and do not change during execution.

☐

- (1 point) For a given problem, a *heuristic* algorithm guarantees optimal solutions.

  Sample solution:

  ☐ True ☒ False

  Explanation: Heuristics are non-optimal algorithms.

☐

3.2: List Scheduling (maximal 13 points)

Consider the sequence graph in Figure 11, and answer the following questions.

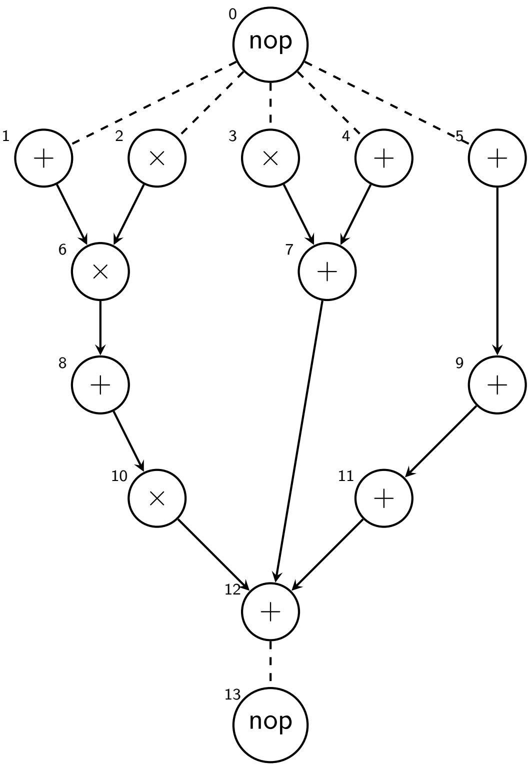


Figure 11: A sequence graph

(a) (10 Points) Suppose that one multiplier and two adders are available as resources. Addition and multiplication take **one** and **two** time units on the adders (r_1, r_2) and multiplier (r_3), respectively. The first operation starts at $t = 0$ and the top node ('nop') is executed in zero time. The priority is assigned for each operation as the *maximal distance* to the bottom node ('nop'). The maximal distance is defined as the number of edges on the longest path between two nodes. Fill out Table 6 using the **List scheduling algorithm**. For a timestep t: $U_{t,k}$ denotes the set of operations that are ready to be scheduled on resource r_k. $S_{t,k}$ denotes the set of operations that start at time t on resource r_k. $T_{t,k}$ denotes the set of operations in execution at time t on resource r_k.

t	k	$U_{t,k}$	$T_{t,k}$	$S_{t,k}$
0	r_1, r_2	ν_1, ν_4, ν_5	—	ν_1, ν_5
	r_3	ν_2, ν_3	—	ν_2
1	r_1, r_2	ν_4, ν_9	—	ν_4, ν_9
	r_3	ν_3	ν_2	—
2	r_1, r_2	ν_{11}	—	ν_{11}
	r_3	ν_3, ν_6	—	ν_6
3	r_1, r_2	—	—	—
	r_3	ν_3	ν_6	—
4	r_1, r_2	ν_8	—	ν_8
	r_3	ν_3	—	ν_3
5	r_1, r_2	—	—	—
	r_3	ν_{10}	ν_3	—
6	r_1, r_2	ν_7	—	ν_7
	r_3	ν_{10}	—	ν_{10}
7	r_1, r_2	—	—	—
	r_3	—	ν_{10}	—
8	r_1, r_2	ν_{12}	—	ν_{12}
	r_3	—	—	—

Table 6: Table for Task 3.2. (a)

(b) (1 Point) What is the corresponding latency?

Sample solution:
$L = 9$.

□

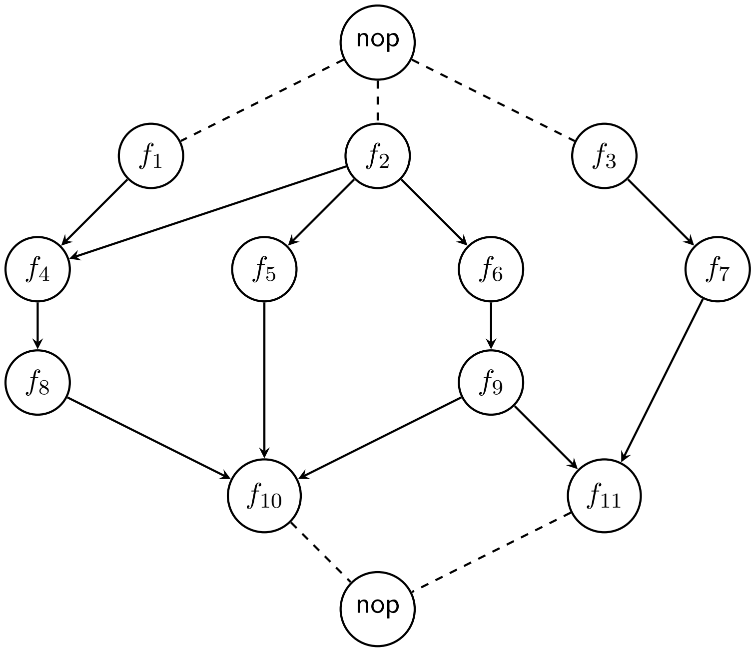
(c) (2 Points) If it is allowed to add a single additional hardware unit, an adder or a multiplier, which resource should be added to reduce the latency? Explain why.

Sample solution:
The critical path $(2 \rightarrow 6 \rightarrow 8 \rightarrow 10 \rightarrow 12)$ bounds the minimum latency. At time $t = 5$ the lack of multipliers introduces a delay. An additional multiplier would avoid this delay, while an additional adder would not reduce the latency.

□

3.3: Timing Constraints (maximal 12 points)

Consider the sequence graph and execution times $w(f_i)$ of tasks f_i in Figure 12, answer the following questions.



Task	$w(f_i)$
f_1	1
f_2	5
f_3	1
f_4	2
f_5	2
f_6	2
f_7	4
f_8	3
f_9	2
f_{10}	3
f_{11}	2

Figure 12: A sequence graph and execution times

(a) (2 Points) Complete the Weighted Constraints Graph $G_C = (V_C, E_C, d)$ by annotating edges with numbers in Figure 12.

Sample solution:
See Figure 13.

□

(b) (5 Points) Formulate the following constraints using $\tau(f_i)$ as the start time of task f_i. Add appropriate annotated edges to the constraint graph G_C in Figure 12.

i) f_2 should start no earlier than 5 time units after the start of f_3.

Sample solution:
$\tau(f_2) \geq \tau(f_3) + 5 \;\Rightarrow\; \tau(f_2) - \tau(f_3) \geq 5$

☐

ii) f_4 should finish no earlier than 3 time units after the start of f_5.

Sample solution:
$\tau(f_4) + 2 \geq \tau(f_5) + 3 \;\Rightarrow\; \tau(f_4) - \tau(f_5) \geq 1$

☐

iii) f_9 should start no later than 8 time units after the finish of f_5.

Sample solution:
$\tau(f_9) \leq \tau(f_5) + 2 + 8 \;\Rightarrow\; \tau(f_5) - \tau(f_9) \geq -10$

☐

iv) f_{10} should start exactly 3 time units after the start of f_{11}.

Sample solution:
$\tau(f_{10}) = \tau(f_{11}) + 3 \;\Rightarrow\; \tau(f_{11}) - \tau(f_{10}) \geq -3 \;\wedge\; \tau(f_{10}) - \tau(f_{11}) \geq 3$

☐

v) f_{10} should finish no later than 14 time units after the finish of f_3.

Sample solution:
$\tau(f_{10}) + 3 \leq \tau(f_3) + 1 + 14 \;\Rightarrow\; \tau(f_3) - \tau(f_{10}) \geq -12$

☐

(c) (5 Points) In case of unlimited resources, does a feasible schedule exist for the given sequence graph and all of the above timing constraints? Justify your answer.

Sample solution:
No. There exist positive cycles in G_C, i.e $f_3, f_2, f_6, f_9, f_{11}, f_{10}, f_3$. See Figure 13.

☐

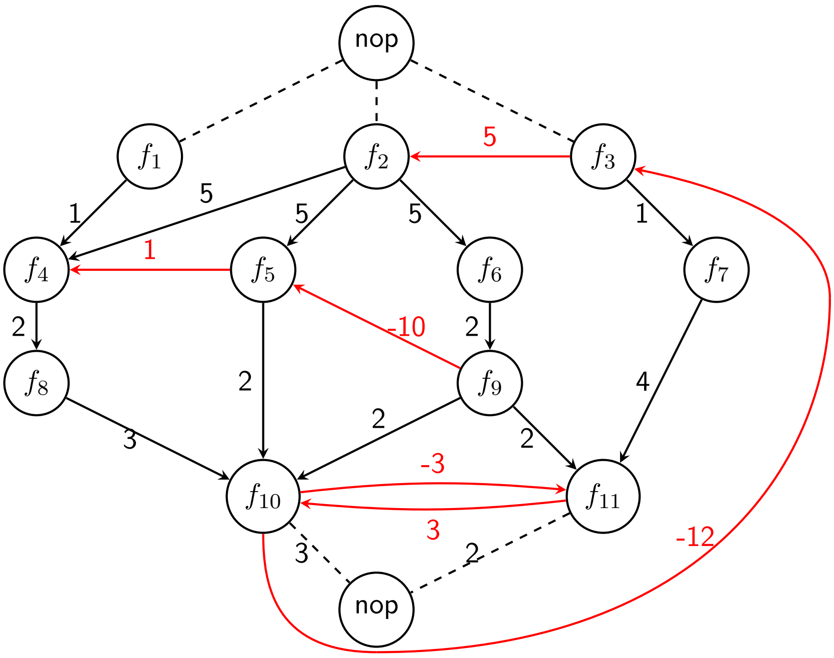
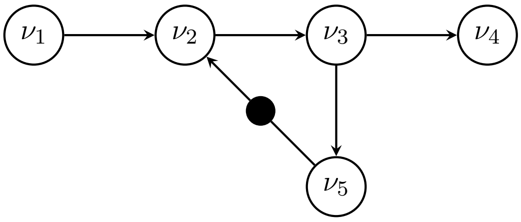


Figure 13: A sequence graph and execution times

3.4: Iterative Algorithm (maximal 9 points)

Consider the marked graph G_M and the task execution times in Figure 14, and answer the following questions.



Task	ν_1	ν_2	ν_3	ν_4	ν_5
$w(\nu_i)$	1	2	1	1	2

Figure 14: A marked graph with task execution times

(a) (5 points) Formulate all existing dependencies in Figure 14 from ν_i to ν_j in the form of

$$\tau(\nu_j) - \tau(\nu_i) \geq w(\nu_i) - d_{ij} \times P,$$

where P is the minimum iteration interval.

Sample solution:
$\tau(\nu_2) - \tau(\nu_1) \geq 1$
$\tau(\nu_3) - \tau(\nu_2) \geq 2$
$\tau(\nu_4) - \tau(\nu_3) \geq 1$
$\tau(\nu_5) - \tau(\nu_3) \geq 1$
$\tau(\nu_2) - \tau(\nu_5) \geq 2 - 1 \times P$

□

(b) (4 Points) Function ν_2 uses the result of ν_5 from the previous iteration. Suppose that any arbitrary number of tokens can be inserted on this edge to reduce P using functional pipelining. Assuming unlimited resources, and that the same operation of different iterations cannot be executed in parallel or overlap, how many tokens should be added on the edge $\nu_5 \rightarrow \nu_2$ to achieve the minimal iteration interval? To justify your answer, draw the pipelined schedule for **four** iterations in Figure 15. Determine the latency L of this schedule.

Sample solution:
Two more tokens should be added. $P = 2$ and $L = 6$. See Figure 15 for the schedule.

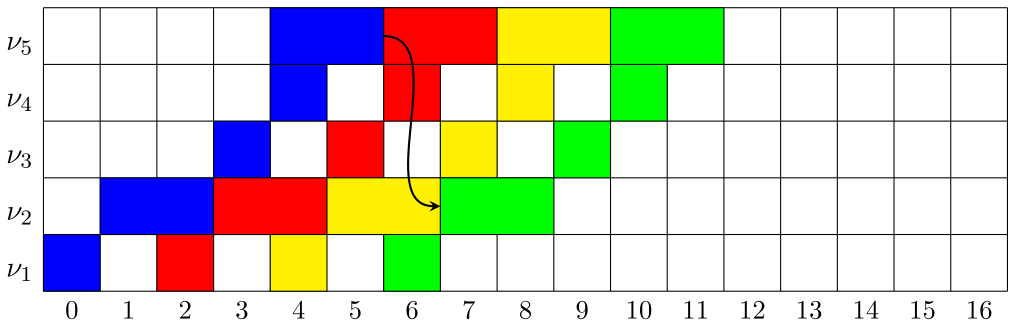


Figure 15: Scheduling grid for Task 3.4 (b)

□